

# A NEW MODEL OF NONLOCAL MODIFIED GRAVITY

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**ABSTRACT.** We consider a new modified gravity model with nonlocal term of the form  $R^{-1}\mathcal{F}(\square)R$ . This kind of nonlocality is motivated by investigation of applicability of a few unusual ansätze to obtain some exact cosmological solutions. In particular, we find attractive and useful quadratic ansatz  $\square R = qR^2$ .

## 1. Introduction

In spite of the great successes of General Relativity (GR) it has not got status of a complete theory of gravity. To modify GR there are motivations coming from its quantum aspects, string theory, astrophysics and cosmology. For example, cosmological solutions of GR contain Big Bang singularity, and Dark Energy as a cause for accelerated expansion of the Universe. This initial cosmological singularity is an evident signature that GR is not appropriate theory of the Universe at cosmic time  $t = 0$ . Also, GR has not been verified at the very large cosmic scale and dark energy has not been discovered in the laboratory experiments. This situation gives rise to research for an adequate modification of GR among numerous possibilities (for a recent review, see [1]).

Recently it has been shown that nonlocal modified gravity with action

$$(1.1) \quad S = \int d^4x \sqrt{-g} \left( \frac{R - 2\Lambda}{16\pi G} + CR\mathcal{F}(\square)R \right),$$

where  $R$  is scalar curvature,  $\Lambda$  – cosmological constant,  $\mathcal{F}(\square) = \sum_{n=0}^{\infty} f_n \square^n$  is an analytic function of the d'Alembert-Beltrami operator  $\square = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$ ,  $g = \det(g_{\mu\nu})$  and  $C$  is a constant, has nonsingular bounce cosmological solutions, see [2, 3, 4, 5]. To solve equations of motion it was used ansatz  $\square R = rR + s$ . In [6] we

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2010 *Mathematics Subject Classification.* Primary 83Dxx, 83Fxx, 53C21; Secondary 83C10, 83C15.

*Key words and phrases.* nonlocal modified gravity, cosmological solutions.

Work partially supported by the Serbian Ministry of Education, Science and Technological Development, contract No. 174012.

introduced some new ansätze, which gave trivial solutions for the above nonlocal model (1.1). In this paper we consider some modification of the above action in the nonlocal sector, i.e.

$$(1.2) \quad S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G} + R^{-1} \mathcal{F}(\square) R \right)$$

and look for nontrivial cosmological solutions for the new ansätze (see [6]). Note that the cosmological constant  $\Lambda$  in (1.2) is hidden in the term  $f_0$ , i.e.  $\Lambda = -8\pi G f_0$ . To the best of our knowledge action (1.2) has not been considered so far. However, there are investigations of gravity modified by  $1/R$  term (see, e.g. [7] and references therein), but it is without nonlocality.

## 2. Equations of motion

By variation of action (1.2) with respect to metric  $g^{\mu\nu}$  one obtains the equations of motion for  $g_{\mu\nu}$

$$(2.1) \quad \begin{aligned} & R_{\mu\nu} V - (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) V - \frac{1}{2} g_{\mu\nu} R^{-1} \mathcal{F}(\square) R \\ & + \sum_{n=1}^{\infty} \frac{f_n}{2} \sum_{l=0}^{n-1} (g_{\mu\nu} (\partial_\alpha \square^l (R^{-1}) \partial^\alpha \square^{n-1-l} R + \square^l (R^{-1}) \square^{n-l} R) \\ & - 2 \partial_\mu \square^l (R^{-1}) \partial_\nu \square^{n-1-l} R) = -\frac{G_{\mu\nu}}{16\pi G}, \\ & V = \mathcal{F}(\square) R^{-1} - R^{-2} \mathcal{F}(\square) R. \end{aligned}$$

The trace of the equation (2.1) is

$$(2.2) \quad \begin{aligned} & R V + 3 \square V + \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} (\partial_\alpha \square^l (R^{-1}) \partial^\alpha \square^{n-1-l} R + 2 \square^l (R^{-1}) \square^{n-l} R) \\ & - 2 R^{-1} \mathcal{F}(\square) R = \frac{R}{16\pi G}. \end{aligned}$$

The 00 component of (2.1) is

$$(2.3) \quad \begin{aligned} & R_{00} V - (\nabla_0 \nabla_0 - g_{00} \square) V - \frac{1}{2} g_{00} R^{-1} \mathcal{F}(\square) R \\ & + \sum_{n=1}^{\infty} \frac{f_n}{2} \sum_{l=0}^{n-1} (g_{00} (\partial_\alpha \square^l (R^{-1}) \partial^\alpha \square^{n-1-l} R + \square^l (R^{-1}) \square^{n-l} R) \\ & - 2 \partial_0 \square^l (R^{-1}) \partial_0 \square^{n-1-l} R) = -\frac{G_{00}}{16\pi G}. \end{aligned}$$

We use Friedmann-Lemaître-Robertson-Walker (FLRW) metric  $ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right)$  and investigate all three possibilities for curvature parameter  $k$  ( $0, \pm 1$ ). In the FLRW metric scalar curvature is  $R = 6 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right)$  and  $\square h(t) = -\partial_t^2 h(t) - 3H \partial_t h(t)$ , where  $H = \frac{\dot{a}}{a}$  is the Hubble parameter. In the sequel we shall use three kinds of ansätze (two of them introduced in [6]) and

solve equations of motions (2.2) and (2.3) for cosmological scale factor in the form  $a(t) = a_0|t - t_0|^\alpha$ .

### 3. Quadratic ansatz: $\square R = qR^2$

Looking for solutions in the form  $a(t) = a_0|t - t_0|^\alpha$  this ansatz becomes

$$(3.1) \quad \alpha(2\alpha - 1)(q\alpha(2\alpha - 1) - (\alpha - 1))(t - t_0)^{-4} + \frac{\alpha k}{3a_0^2}(1 - \alpha + 6q(2\alpha - 1))(t - t_0)^{-2\alpha-2} + \frac{qk^2}{a_0^4}(t - t_0)^{-4\alpha} = 0.$$

Equation (3.1) is satisfied for all values of time  $t$  in six cases:

$$\begin{aligned} (1) \quad & k = 0, \alpha = 0, q \in \mathbb{R}, & (4) \quad & k = -1, \alpha = 1, q \neq 0, a_0 = 1, \\ (2) \quad & k = 0, \alpha = \frac{1}{2}, q \in \mathbb{R}, & (5) \quad & k \neq 0, \alpha = 0, q = 0, \\ (3) \quad & k = 0, \alpha \neq 0 \text{ and } \alpha \neq \frac{1}{2}, & (6) \quad & k \neq 0, \alpha = 1, q = 0. \\ & q = \frac{\alpha-1}{\alpha(2\alpha-1)}, \end{aligned}$$

In the cases (1), (2) and (4) we have  $R = 0$  and therefore  $R^{-1}$  is not defined. The case (5) yields a solution which does not satisfy equations of motion. Hence there remain two cases for further consideration.

**3.1. Case  $k = 0, q = \frac{\alpha-1}{\alpha(2\alpha-1)}$ .** For this case, we have the following expressions depending on the parameter  $\alpha$ :

$$(3.2) \quad \begin{aligned} q &= \frac{\alpha - 1}{\alpha(2\alpha - 1)}, & R &= 6\alpha(2\alpha - 1)(t - t_0)^{-2}, \\ a &= a_0|t - t_0|^\alpha, & H &= \alpha(t - t_0)^{-1}, \\ R_{00} &= 3\alpha(1 - \alpha)(t - t_0)^{-2}, & G_{00} &= 3\alpha^2(t - t_0)^{-2}. \end{aligned}$$

We now express  $\square^n R$  and  $\square^n R^{-1}$  in the following way:

$$(3.3) \quad \begin{aligned} \square^n R &= B(n, 1)(t - t_0)^{-2n-2}, \quad \square^n R^{-1} = B(n, -1)(t - t_0)^{2-2n}, \\ B(n, 1) &= 6\alpha(2\alpha - 1)(-2)^n n! \prod_{l=1}^n (1 - 3\alpha + 2l), \quad n \geq 1, \\ B(n, -1) &= (6\alpha(2\alpha - 1))^{-1} 2^n \prod_{l=1}^n (2 - l)(-3 - 3\alpha + 2l), \quad n \geq 1, \\ B(0, 1) &= 6\alpha(2\alpha - 1), \quad B(0, -1) = B(0, 1)^{-1}. \end{aligned}$$

Note that  $B(1, -1) = -\frac{3\alpha+1}{3\alpha(2\alpha-1)} = -2(3\alpha+1)B(0, 1)^{-1}$  and  $B(n, -1) = 0$  if  $n \geq 2$ . Also, we obtain

$$(3.4) \quad \begin{aligned} \mathcal{F}(\square)R &= \sum_{n=0}^{\infty} f_n B(n, 1)(t - t_0)^{-2n-2}, \\ \mathcal{F}(\square)R^{-1} &= f_0 B(0, -1)(t - t_0)^2 + f_1 B(1, -1). \end{aligned}$$

Substituting these equations into trace and 00 component of the EOM one has

$$\begin{aligned}
& r^{-1} \sum_{n=0}^{\infty} f_n B(n, 1) (-3r + 6(1-n)(1-2n+3\alpha)) (t-t_0)^{-2n} \\
& + r \sum_{n=0}^1 f_n (rB(n, -1) + 3B(n+1, -1)) (t-t_0)^{-2n} \\
& + 2r \sum_{n=1}^{\infty} f_n \gamma_n (t-t_0)^{-2n} = \frac{r^2}{16\pi G} (t-t_0)^{-2}, \\
(3.5) \quad & \sum_{n=0}^{\infty} f_n r^{-1} B(n, 1) \left( \frac{r}{2} - A_n \right) (t-t_0)^{-2n} \\
& + \sum_{n=0}^1 f_n r B(n, -1) A_n (t-t_0)^{-2n} + \frac{r}{2} \sum_{n=1}^{\infty} f_n \delta_n (t-t_0)^{-2n} \\
& = \frac{-r^2}{32\pi G} \frac{\alpha}{2\alpha-1} (t-t_0)^{-2},
\end{aligned}$$

where  $r = B(0, 1)$  and

$$\begin{aligned}
(3.6) \quad \gamma_n &= \sum_{l=0}^{n-1} B(l, -1) (B(n-l, 1) + 2(1-l)(n-l)B(n-l-1, 1)), \\
\delta_n &= \sum_{l=0}^{n-1} B(l, -1) (-B(n-l, 1) + 4(1-l)(n-l)B(n-l-1, 1)), \\
A_n &= 6\alpha(1-n) - r \frac{\alpha-1}{2(2\alpha-1)} = \frac{r}{2} \frac{3-2n-\alpha}{2\alpha-1}.
\end{aligned}$$

Equations (3.5) can be split into system of pairs of equations with respect to each coefficient  $f_n$ . In the case  $n > 1$ , there are the following pairs:

$$\begin{aligned}
(3.7) \quad & f_n (B(n, 1) (-3r + 6(1-n)(1-2n+3\alpha)) + 2r^2 \gamma_n) = 0, \\
& f_n \left( B(n, 1) \left( \frac{r}{2} - A_n \right) + \frac{r^2}{2} \delta_n \right) = 0.
\end{aligned}$$

Taking  $\frac{3\alpha-1}{2}$  to be a natural number one obtains:

$$(3.8) \quad B(n, 1) = 6\alpha(2\alpha-1)4^n n! \frac{(\frac{3}{2}(\alpha-1))!}{(\frac{3}{2}(\alpha-1)-n)!}, \quad n < \frac{3\alpha-1}{2},$$

$$(3.9) \quad B(n, 1) = 0, \quad n \geq \frac{3\alpha-1}{2},$$

$$(3.10) \quad \gamma_n = 2B(0, -1)B(n-1, 1)(3n\alpha - 2n^2 - 3\alpha - 1), \quad n \leq \frac{3\alpha-1}{2},$$

$$(3.11) \quad \delta_n = 2B(0, -1)B(n-1, 1)(2n^2 + 3n + 3\alpha - 3\alpha n + 1), \quad n \leq \frac{3\alpha-1}{2},$$

$$(3.12) \quad \gamma_n = \delta_n = 0, \quad n > \frac{3\alpha-1}{2}.$$

If  $n > \frac{3\alpha-1}{2}$  then  $B(n, 1) = \gamma_n = \delta_n = 0$  and hence the system is trivially satisfied for arbitrary value of coefficients  $f_n$ . On the other hand for  $2 \leq n \leq \frac{3\alpha-1}{2}$  the system has only trivial solution  $f_n = 0$ .

When  $n = 0$  the pair becomes

$$(3.13) \quad f_0(-2r + 6(1 + 3\alpha) + 3rB(1, -1)) = 0, \quad f_0 = 0$$

and its solution is  $f_0 = 0$ . The remaining case  $n = 1$  reads

$$(3.14) \quad \begin{aligned} f_1(-3r^{-1}B(1, 1) + rB(1, -1) + 2\gamma_1) &= \frac{r}{16\pi G}, \\ f_1\left(A_1(rB(1, -1) - r^{-1}B(1, 1)) + \frac{1}{2}(B(1, 1) + r\delta_1)\right) &= \frac{-r^2}{32\pi G} \frac{\alpha}{2\alpha - 1}, \end{aligned}$$

and it gives  $f_1 = -\frac{3\alpha(2\alpha-1)}{32\pi G(3\alpha-2)}$ .

**3.2. Case  $k \neq 0$ ,  $\alpha = 1$ ,  $q = 0$ .** In this case

$$(3.15) \quad \begin{aligned} a &= a_0|t - t_0|, \quad H = (t - t_0)^{-1}, \quad R = s(t - t_0)^{-2}, \\ s &= 6(1 + \frac{k}{a_0^2}), \quad \square R = 0, \quad R_{00} = 0, \\ \square^n R^{-1} &= D(n, -1)(t - t_0)^{2-2n}, \\ D(0, -1) &= s^{-1}, \quad D(1, -1) = -8s^{-1}, \quad D(n, -1) = 0, \quad n \geq 2. \end{aligned}$$

Substitution of the above expressions in trace and 00 component of the EOM yields

$$(3.16) \quad \begin{aligned} 3f_0 + \sum_{n=0}^1 f_n s D(n, -1)(t - t_0)^{-2n} + 4f_1(t - t_0)^{-2} &= \frac{s}{16\pi G}(t - t_0)^{-2}, \\ -6f_0 s^{-1} + \frac{1}{2}f_0 + 6 \sum_{n=0}^1 f_n D(n, -1)(1 - n)(t - t_0)^{-2n} \\ + 2f_1(t - t_0)^{-2} &= -\frac{s}{32\pi G}(t - t_0)^{-2}. \end{aligned}$$

This system leads to conditions for  $f_0$  and  $f_1$  :

$$(3.17) \quad \begin{aligned} -2f_0 - 4f_1(t - t_0)^{-2} &= \frac{s}{16\pi G}(t - t_0)^{-2}, \\ \frac{1}{2}f_0 + 2f_1(t - t_0)^{-2} &= -\frac{s}{32\pi G}(t - t_0)^{-2}. \end{aligned}$$

The corresponding solution is

$$(3.18) \quad f_0 = 0, \quad f_1 = \frac{-s}{64\pi G}, \quad f_n \in \mathbb{R}, \quad n \geq 2.$$

**4. Ansatz**  $\square^n R = c_n R^{n+1}$ ,  $n \geq 1$ 

Presenting  $\square^{n+1} R$  in two ways:

$$\begin{aligned}
 \square^{n+1} R &= \square c_n R^{n+1} \\
 &= c_n ((n+1) R^n \square R - n(n+1) R^{n-1} \dot{R}^2) \\
 &= c_n (n+1) (c_1 R^{n+2} - n R^{n-1} \dot{R}^2) \\
 &= c_{n+1} R^{n+2}
 \end{aligned}$$

it follows

$$(4.1) \quad \dot{R}^2 = R^3,$$

$$(4.2) \quad c_{n+1} = c_n (n+1) (c_1 - n),$$

where  $\dot{R}^2$  means  $(\dot{R})^2$ .

General solution of equation (4.1) is

$$(4.3) \quad R = \frac{4}{(t - t_0)^2}, \quad t_0 \in \mathbb{R}.$$

Taking  $n = 1$  in the ansatz yields

$$(4.4) \quad \square R = c_1 R^2.$$

Substitution of (4.3) in (4.4) gives  $H = \frac{2c_1+3}{3(t-t_0)}$ . This implies

$$(4.5) \quad a(t) = a_0 |t - t_0|^{\frac{2c_1+3}{3}}, \quad a_0 > 0.$$

Using (4.3) in equation

$$(4.6) \quad R = 6 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right)$$

gives

$$(4.7) \quad (t - t_0)^2 \ddot{y} - \frac{4}{3} y = -2k(t - t_0)^2, \quad \text{where } y = a^2(t).$$

It can be shown that general solution of the last equation is

$$(4.8) \quad a^2(t) = \tilde{C}_1 |t - d_1|^{\frac{3+\sqrt{57}}{6}} + \tilde{C}_2 |t - d_1|^{\frac{3-\sqrt{57}}{6}} - 3k|t - d_1|^2, \quad \tilde{C}_1, \tilde{C}_2 \in \mathbb{R}.$$

By comparison of the last equation with (4.5) one can conclude:

- (1) If  $c_1 = 0$  then  $k$  must be equal to  $-1$ . In this case  $\square^n R = 0$ ,  $n \geq 1$ .
- (2) If  $c_1 \neq 0$  then  $k$  must be equal to  $0$ . In this case  $c_1 = \frac{-9 \pm \sqrt{57}}{8}$ .

**4.1. Case  $\square^n R = c_n R^{n+1}$ ,  $c_1 = 0$ .** From the previous analysis, it follows:

$$(4.9) \quad \begin{aligned} k &= -1, & a(t) &= \sqrt{3}|t - t_0|, & H(t) &= \frac{1}{t - t_0}, \\ R &= \frac{4}{(t - t_0)^2}, & \square^n R &= 0, \quad n \geq 1, & \mathcal{F}(\square)R &= f_0 R. \end{aligned}$$

It can be shown that

$$(4.10) \quad \square^n R^{-1} = (-1)^n 4^{n-1} \prod_{l=0}^{n-1} (1-l)(2-l)(t-t_0)^{2-2n}.$$

From (4.10) follows  $\square^n R^{-1} = 0$ ,  $n > 1$ . Then

$$(4.11) \quad \mathcal{F}(\square)(R^{-1}) = f_0 R^{-1} + f_1 \square R^{-1}.$$

Substituting (4.9) and (4.11) in the 00 component of the EOM one obtains

$$(4.12) \quad \frac{f_0}{2}(t-t_0)^2 + 2f_1 + \frac{1}{8\pi G} = 0$$

and it follows

$$(4.13) \quad f_0 = 0, \quad f_1 = \frac{-1}{16\pi G}, \quad f_n \in \mathbb{R}, \quad n \geq 2.$$

Substituting (4.9) and (4.11) in the trace equation one has

$$(4.14) \quad -2f_0(t-t_0)^2 - 4f_1 - \frac{1}{4\pi G} = 0$$

and it gives the same result (4.13).

**4.2. Case  $\square^n R = c_n R^{n+1}$ ,  $c_1 = \frac{-9 \pm \sqrt{57}}{8}$ .** In this case:

$$(4.15) \quad \begin{aligned} k &= 0, & R &= \frac{4}{(t-t_0)^2}, & H &= \frac{2c_1+3}{3(t-t_0)}, & a &= a_0|t-t_0|^{\frac{2c_1+3}{3}}, & a_0 &> 0, \\ R_{00} &= 3\alpha(1-\alpha)(t-t_0)^{-2}, & G_{00} &= (3\alpha(1-\alpha)+2)(t-t_0)^{-2}, & \alpha &= \frac{2c_1+3}{3}, \\ \square^n R &= 4^{n+1}c_n(t-t_0)^{-2n-2}, & c_0 &= 1. \end{aligned}$$

One can show that

$$(4.16) \quad \square^n R^{-1} = M(n, -1)(t-t_0)^{2-2n},$$

where

$$(4.17) \quad M(0, -1) = \frac{1}{4}, \quad M(1, -1) = -(c_1 + 2), \quad M(n, -1) = 0, \quad n > 1.$$

Also one obtains

$$(4.18) \quad \begin{aligned} \mathcal{F}(\square)R &= \sum_{n=0}^{\infty} 4^{n+1}f_n c_n (t-t_0)^{-2n-2}, \\ \mathcal{F}(\square)R^{-1} &= f_0 M(0, -1)(t-t_0)^2 + f_1 M(1, -1). \end{aligned}$$

Substituting (4.18) in the trace equation it becomes

$$\begin{aligned}
& -\frac{1}{4\pi G}(t-t_0)^{-2} - 2f_0 - 3\sum_{n=1}^{\infty} 4^n f_n c_n (t-t_0)^{-2n} \\
(4.19) \quad & + \sum_{n=1}^{\infty} f_n \left( \sum_{l=0}^{n-1} M(l, -1) 4^{n-l+1} ((1-l)(n-l)c_{n-1-l} + 2c_{n-l}) \right) (t-t_0)^{-2n} \\
& + f_1 (4M(1, -1) + 3M(2, -1)) (t-t_0)^{-2} = 0.
\end{aligned}$$

To satisfy equation (4.19) for all values of time  $t$  one obtains:

$$(4.20) \quad f_0 = 0, \quad f_1(2c_1 + 1) = -\frac{1}{16\pi G},$$

$$(4.21) \quad f_n \left( -3c_n + \sum_{l=0}^1 M(l, -1) 4^{1-l} ((1-l)(n-l)c_{n-1-l} + 2c_{n-l}) \right) = 0, \quad n \geq 2.$$

Suppose that  $f_n \neq 0$  for  $n \geq 2$ , then from the last equation follows

$$(4.22) \quad -3c_n + \sum_{l=0}^1 M(l, -1) 4^{1-l} ((1-l)(n-l)c_{n-1-l} + 2c_{n-l}) = 0$$

and it becomes

$$(4.23) \quad c_{n-1}(n^2 - c_1 n - 2c_1 - 4) = 0.$$

Since  $c_{n-1} \neq 0$ , condition (4.23) is satisfied for  $n = -2$  or  $n = c_1 + 2$ . Hence, we conclude that  $f_n = 0$  for  $n \geq 2$ .

Since  $f_n = 0$  for  $n \geq 2$ , the 00 component of the EOM becomes

$$\begin{aligned}
(4.24) \quad & \frac{1}{16\pi G}(-3\alpha^2 + 3\alpha + 2)(t-t_0)^{-2} \\
& + \frac{1}{2}f_0\left(\frac{3}{2}\alpha^2 - \frac{9}{2}\alpha + 1\right) + f_1 c_1 (3\alpha^2 - 3\alpha + 2)(t-t_0)^{-2} \\
& + 8f_1 M(0, -1)(1-c_1)(t-t_0)^{-2} + 3\alpha(3-\alpha)M(0, -1)f_0 \\
& - 3\alpha(\alpha-1)M(1, -1)f_1(t-t_0)^{-2} = 0.
\end{aligned}$$

In order to satisfy equation (4.24) for all values of time  $t$  it has to be

$$(4.25) \quad f_0 = 0, \quad f_1 \left( \frac{4}{3}c_1^3 + \frac{10}{3}c_1^2 + 2c_1 + 1 \right) = \frac{1}{16\pi G} \left( \frac{2}{3}c_1^2 + c_1 - 1 \right).$$

The necessary and sufficient condition for the EOM to have a solution is

$$(4.26) \quad c_1(8c_1^2 + 18c_1 + 3) = 0.$$

Since  $c_1 = \frac{-9 \pm \sqrt{57}}{8}$ , the last condition is satisfied.



### 5. Cubic ansatz: $\square R = qR^3$

Recall that we are looking for solutions in the form  $a(t) = a_0|t - t_0|^\alpha$ . In the explicit form it reads

$$(5.1) \quad \begin{aligned} & \alpha(\alpha - 1) \left( 3(2\alpha - 1)(t - t_0)^{-4} + \frac{k}{a_0^2}(t - t_0)^{-2\alpha-2} \right) \\ & = 18q \left( \alpha(2\alpha - 1)(t - t_0)^{-2} + \frac{k}{a_0^2}(t - t_0)^{-2\alpha} \right)^3. \end{aligned}$$

It yields the following seven possibilities:

- |  |  |
|--|--|
| (1) $k = 0, \alpha = 0, q \in \mathbb{R},$           | (5) $k \neq 0, \alpha = 0, q = 0,$                           |
| (2) $k = 0, \alpha = \frac{1}{2}, q \in \mathbb{R},$ | (6) $k \neq 0, \alpha = 1, q = 0,$                           |
| (3) $k = -1, \alpha = 1, q \neq 0, a_0 = 1,$         | (7) $k \neq 0, \alpha = \frac{1}{2}, q = -\frac{a_0^4}{72}.$ |
| (4) $k = 0, \alpha = 1, q = 0,$                      |  |

Cases (1), (2) and (3) contain scalar curvature  $R = 0$ , and therefore we will not discuss them. Cases (4), (5) and 6 are also obtained from the quadratic ansatz and have been discussed earlier. The last case contains:

$$(5.2) \quad \begin{aligned} a(t) &= a_0 \sqrt{|t - t_0|}, \quad H(t) = \frac{1}{2(t - t_0)}, \\ R(t) &= \frac{6k}{a_0^2} |t - t_0|^{-1}, \quad R_{00} = \frac{3}{4(t - t_0)^2}. \end{aligned}$$

One can derive the following expressions:

$$(5.3) \quad \begin{aligned} \square^n R &= N(n, 1)|t - t_0|^{-2n-1}, \quad \square^n R^{-1} = N(n, -1)|t - t_0|^{1-2n}, \\ N(0, 1) &= \frac{6k}{a_0^2}, \quad N(0, -1) = N(0, 1)^{-1}, \\ N(n, 1) &= N(0, 1)(-1)^n \prod_{l=0}^{n-1} (2l + 1)(2l + \frac{1}{2}), \quad n \geq 1, \\ N(n, -1) &= N(0, 1)^{-1}(-1)^n \prod_{l=0}^{n-1} (2l - 1)(2l - \frac{3}{2}), \quad n \geq 1, \\ \mathcal{F}(\square)R &= \sum_{n=0}^{\infty} f_n N(n, 1)|t - t_0|^{-2n-1}, \\ \mathcal{F}(\square)R^{-1} &= \sum_{n=0}^{\infty} f_n N(n, -1)|t - t_0|^{1-2n}. \end{aligned}$$

Substituting (5.3) in the trace equation we obtain

$$\begin{aligned}
(5.4) \quad & -2N(0,1)^{-1} \sum_{n=0}^{\infty} f_n N(n,1) |t-t_0|^{-2n} + N(0,1) \sum_{n=0}^{\infty} f_n (N(n,-1) \\
& - N(0,1)^{-2} N(n,1)) |t-t_0|^{-2n} \\
& + 3 \sum_{n=0}^{\infty} f_n (N(n,-1) - N(0,1)^{-2} N(n,1)) |t-t_0|^{-1-2n} \\
& + \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} N(l,-1) ((1-2l)(-2n+2l+1) N(n-l-1,1) \\
& + 2N(n-l,1)) |t-t_0|^{-2n} = \frac{N(0,1)}{16\pi G} |t-t_0|^{-1}.
\end{aligned}$$

This equation implies the following conditions on coefficient  $f_0$  :

$$(5.5) \quad f_0 = 0, \quad \frac{N(0,1)}{16\pi G} = 0.$$

Since  $N(0,1) \neq 0$ , the last equation never holds and therefore there is no solution in this case.

## 6. Concluding remarks

Using a few new ansätze we have shown that equations of motion for nonlocal gravity model given by action (1.2) yield some bounce cosmological solutions of the form  $a(t) = a_0 |t-t_0|^\alpha$ . These solutions lead to  $f_0 = 0$  and hence  $\Lambda = 0$ , and when  $t \rightarrow \infty$  then  $R \rightarrow 0$ . In particular, quadratic ansatz  $\square R = qR^2$  is very promising. Note that ansatz  $\square^n R = c_n R^{n+1}$ ,  $n \geq 1$ , can be viewed as a special case of ansatz  $\square R = qR^2$ .

It is worth noting that equations of motion (2.2) and (2.3) have the de Sitter solutions  $a(t) = a_0 \cosh(\lambda t)$ ,  $k = +1$  and  $a(t) = a_0 e^{\lambda t}$ ,  $k = 0$ , when  $f_0 = \frac{-3\lambda^2}{8\pi G} = \frac{-\Lambda}{8\pi G}$ ,  $f_n \in \mathbb{R}$ ,  $n \geq 1$ .

This investigation can be generalized to some cases with  $R^{-p} \mathcal{F}(\square) R^q$  nonlocal term, where  $p$  and  $q$  are some natural numbers satisfying  $q - p \geq 0$ . It will be presented elsewhere with discussion of various properties.

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